Big Bang Nucleosynthesis Lab - NS102

May 23, 2005

1 Introduction

This week we will be ignoring the universe's first 0.001 seconds, and learning about the Era of Nucleosynthesis, when the soup of protons and some neutrons began the approximately three-minute long process of fusion into heavier elements.¹

2 The Elemental Guide to Elements

FUSION

Fusion is the process of two atoms banging together to make a heavier atom in such a way that energy is released. (It's how the sun works and how hydrogen bombs work.) The reaction is written in shorthand as:

$$A + B \longrightarrow C + \gamma$$
 (1)

where the gamma can be considered a "photon", a particle of light energy. Product C of this collision has slightly less mass than the masses of A and B added together. The difference in the masses is equal to the energy of the photon, or to put it in math language:

$$m_C - (m_A + m_B) = E_\gamma / c^2 \tag{2}$$

In other words, $E = \Delta m \times c^2$, something you've probably seen before. (Recall that c is the speed of light.) The next problem should make you feel like Einstein.

1. How much energy does the mass of a penny represent? There is a scale in the lab. Put the answer you get into kilograms and use the speed of light in meters per second. Your answer will automatically be in the right energy units (call them "Joules"). For comparison, a hairdryer uses about 1,000 Watts, which means it uses 1000 Joules every second. The Hoover Dam produces about 2 Gigawatts = $2 \times 10^9 J/s$ so that the Las Vegas Strip can stay all lit up. Your answer should show that a penny's mass is equal to over a day's output of energy from the Hoover dam!

¹Heavy in this context is actually quite light. If you look at a chemical table of elements, only the first six or seven elements are worth following in this drama. Anything heavier will be such an infinitessimal fraction of the final primordial abundance as to not warrant further discussion in this class.

PROTONS

The simplest element in the universe is Hydrogen which is made up of one proton and an electron. For today's lab, the temperature of our toy universe is so hot that **electrons** will be **ignored**...anytime a proton manages to capture an electron, it would be knocked away by the high-energy radiation (photons) almost as fast, so we'll completely disregard them. So our hydrogen is just a proton which today is a thick tinkertoy disc with holes along the edge and the center.

$$NEUTRON + PROTON = DEUTERON + \gamma$$

If a proton and neutron manage to fuse, you still have a hydrogen-like atom (it's the number of protons that determine this: one proton regardless of the number of neutrons means hydrogen-like, two means helium-like). But it's roughly twice as heavy. We call this creation a deuteron. Today, the rôle of Neutrons will be played by the connector cylinders with a hole at either end and one through the middle.

$$NEUTRON + DEUTERON = TRITIUM + \gamma$$

If a deuteron manages to combine with a neutron, you still have a hydrogen like atom, just (roughly) three times as heavy.

$$PROTON + DEUTERON = HELIUM-3 + \gamma$$

But if a deuteron combines with a proton, you get a helium-like atom (since there are two protons), but not as heavy as the stable Helium-4 atom you find in balloons at parties.

PROTON + TRITIUM = HELIUM-4 +
$$\gamma$$

NEUTRON + HELIUM-3 = HELIUM-4 + γ

Finally, if either of the above reactions occur you get a Helium-4 (standard Helium) which is very stable. For today, we will use the symbols p for proton, n for neutron, d for deuteron, T for tritium, 3He for Helium-3 and 4He for Helium-4. And we will only allow the above reactions.

BACKREACTIONS
$$(\gamma + C \longrightarrow A + B)$$

The universe was still hot enough during the early part of those three minutes that "heavy" elements were broken apart by the surrounding radiation almost as often as they were created. Rods of any color will be double cast in our drama as either bonds **or** as loose radiation ("photons") running around trying to break things up.

(Technically, the cross-section (a measure of how likely two particles are to fuse if they come near each other) for creation was on the same order as the cross-section for an already existing fused element to break apart.)

The result of all this is that after a settling in period, the various elements would fall into nuclear statistical equilibrium, meaning the actual number of each species/element

remains roughly constant as a function of time. In our toy universe, remember, we will not complicate things with any element heavier than Helium-4.

In Chemistry and Physics, equilibrium is usually indicated by making arrows for a reaction point both ways, so our five possible reactions look like this (the numbers to the right will be used later):

$$p+n \rightleftharpoons d+\gamma \qquad 7$$

$$p+d \rightleftharpoons {}^{3}He+\gamma \qquad 5$$

$$n+d \rightleftharpoons T+\gamma \qquad 6$$

$$p+T \rightleftharpoons {}^{4}He+\gamma \qquad 4$$

$$n+{}^{3}He \rightleftharpoons {}^{4}He+\gamma \qquad 5$$

Freezeout

As the universe cooled through expansion, the creation of heavier elements became statistically favored over their destruction; eventually, each heavier species would experience freezeout, when their destruction was far less favorable than their creation. We will model this by suddenly disallowing the breakup of certain elements.

Eventually, as the universe continues to cool, even the likelihood of creating heavier elements freezes out and you are left with the *primordial abundance* of heavy elements. So there are two time-scales in competition with each other: the rates at which heavy elements are created and how fast the universe is cooling.

Given measured abundances of Helium (and some Lithium and Deuterons), we can infer the density of matter and from that we can infer that the universe cooled within three minutes to a point where nucleosynthesis stops. How can we infer this?

- If the universe cools too slowly, *i.e.*, stays hot longer, then freezeout takes too long to occur and fusion can continue on to heavier and heavier elements (leaving us with fewer and fewer light elements, by the way).
- If the universe cools faster we'd never get the chance to make as much Helium and the amount of deuterium observed would be larger.
- If it cools way too fast, neutrons wouldn't have the chance to combine at all, and we'd have no heavy elements.

This last case is a real problem for neutrons because outside an atomic nucleus, a free neutron would just decay spontaneously in about 10 minutes:

$$n \longrightarrow p + e^-$$
 (+ some other stuff) (4)

so we'd have no heavy elements at all. For our tinkertoy exercises, we'll ignore neutron decay (a reasonable assumption in any case for rough calculations).

You'll note in textbooks that after big bang nucleosynthesis is done about 75% of the universe by weight is made up of Hydrogen (protons) and about 25% by weight is made of Helium (two protons and two neutrons stuck together). Assuming protons and neutrons weigh roughly the same, and ignoring the decay of free neutrons, this already tells us the percentage of neutrons there must have been at the beginning of Nucleosynthesis.

2. Assume that you are told that the measured primordial abundance of Helium is 25% by weight, the rest is made up of protons by weight. How many protons were there for every neutron before fusion started? Hint: believe it or not, it helps to draw pictures...

You will notice numbers next to each reaction in the list of reactions above. These will be used for probabilities later; they are NOT based on the actual reaction rates, but were redesigned for our toy universe. The "real" calculation is very similar in process to what we're about to do, but involves many more possible reactions which we will ignore. Have someone in the group rewrite the chart on the board so that we can all see it easily before the fun begins...

3 Supplies

- \cdot Spools (the round tinkertoy discs with holes along the edge) = protons
- · Connectors (the long tinkertoy cylinders) = neutrons
- \cdot Rods (any color) = either bonds or photons
- · Probability Generators (dice)

4 Nucleosynthesis According to Hoyle

The above title is a very bad joke. There was a physicist named Fred Hoyle who did pioneering work on nucleosynthesis in the mid-20th century; he did not write the games book which bears the same name.

These games will be done with the whole group. At any given moment, each person will be responsible for some particles which could be any combination of the following: proton, neutron, deuteron, tritium, helium-3, helium-4 or photon. YOU SHOULD ALSO HAVE A DIE (singular of dice) ON YOU AT ALL TIMES. Connecting rods are placed around the room for easy convenience when needed.

Cosmic Soup

The TA will give each of you some initial protons and neutrons and rearrange you in the room into a primordial cosmic soup. When the TA calls "go!" you start walking around in a random fashion. When the TA says "stop!", you must stop and interact with the person nearest you (but only if you are also the nearest one to them!). The TA will sort out any confusion. YOU MAY ONLY INTERACT ONCE at each round. After everyone has done their thang, the TA will call "go!" again and you start walking around the room again. Try to walk around randomly, and not simply follow your favorite pretty face.

Interactions

Look at what each of you has and the list on the board and figure out which interactions are possible using one item from each person. If there aren't any nothing happens that turn. If only a single interaction is possible that is what you must try for. If multiple

interactions are possible roll your dice - the highest roller gets to decide which interaction you will try for. (If you get the same number roll again until you get different numbers.)

Once you've decided on what your interaction will be roll your dice again. If the total number is **equal to or less than the number** next to the interaction on the board the interaction succeeds. Pick up a bond and a photon if you make something. Put down the bond and the photon if you break something. The person who rolled the higher number gets to decide which of the products they want to keep.

FREEZEOUT

After a certain number of interactions, the TA will call "freezeout". At this point, backreactions are no longer allowed (but forward ones still will be). If you have nothing left but photons and/or Helium-4 you are all done. Step aside and relax while the game plays out.

The game continues for each person until no further interactions are possible, or until the TA calls "second freezeout" again, ending the game. At this point, write your name on the board and list what you have left and how many of each thing. DO NOT LIST PHOTONS. Write down the complete list in your notes.

The TA will collect all the particles from each of you, verifying what you wrote on the board and get it all ready for a new round if there's time. In the meantime, ponder the following questions:

- **3.** Adding up everything on the board, how many neutrons did we start with? What was the initial neutron to proton ratio?
- 4 If you started with a 1:1 ratio of neutrons to protons, and followed the rules of our game what would the outcome be?
- **5** If you started big bang nucleosynthesis with just neutrons, would anything happen? How?

Your write up should include the answers to the boxed questions 1 to 5.