The Lateral Distribution of Extensive Air Showers Produced by Cosmic Rays above 10^{19} eV as Measured by Water-Čerenkov Detectors

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Measurements of the lateral distribution function (ldf) of Extensive Air Showers (EAS) as recorded by the array of water-Čerenkov detectors at Haverah Park are described, and accurate experimental parameterizations expressing the mean ldf for $2 \times 10^{17} < E < 4 \times 10^{18}$ eV, $50 < r < 700$ m, and $\theta < 45°$ are given. An extrapolation of these relations to the regime $E \geq 10^{19}$ eV and $r > 700$ m is described: extrapolation in this energy domain appears valid, and an approximate correction term is given for the larger core distances. The results of recent Monte Carlo simulations of shower development and detector behavior are compared to the parameterized ldf. The agreement is good increasing confidence that these simulations may be trusted as design tools for the Auger project, a proposed "next generation" detector system.

1 Introduction

Planning is currently under way for the construction of two giant detectors, each of order 3000 km^2, to measure the properties of cosmic rays above 10^{19} eV from sites in the Northern and Southern Hemispheres with high statistics. Amongst the conclusions from a study completed at Fermi Lab in July 1995 [1] is the recommendation that the particle detectors for the shower array be water-Čerenkov counters, similar to those of the Haverah Park array [2], the only array which has used such detectors to investigate cosmic rays at the relevant energies. In this paper we describe data recorded at Haverah Park

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over a decade ago which are of particular relevance to the Auger project and which have not hitherto been readily available [3,4]. In addition these data are compared with recent Monte Carlo simulations of giant air showers.

The Auger collaboration has adopted water-Čerenkov detectors, so far used uniquely at the Haverah Park array, as the device of choice for the ground array aspect of the project. The decision was made for essentially the same reasons as drove the original selection. The detectors are 1.2 m deep and so absorb almost completely the electromagnetic component of the shower cascade which has a low average energy (a few MeV) at the relevant distances from the axis of the shower. Muons however penetrate the full depth of the water-tanks and separation of signal amplitudes in the time domain using a flash ADC may provide a powerful method of identifying the fraction of muons in the shower, possibly an important component of methods which will be used to derive the mass composition of the primaries. The separation ability of gross time structure parameters (such as rise-time) is also considerably enhanced for a water-Čerenkov detector due to the significant signal fraction contributed by the early arriving shower muons. In addition the large depth of the detectors and the sensitivity of them to muons permits showers to be detected at large zenith angle at a useful rate much more effectively than would an array of scintillators so that all sky coverage, essential for discovering arrival direction anisotropies, can be achieved effectively with two observatories.

Early experimental studies at Haverah Park by Kellerman and Towers [5] showed that the mean energy of the electrons and photons at about 500 m from the axis of showers of 10^{17} eV was ~ 10 MeV and that the photons outnumber the electrons by a factor of 10 at this distance and by an increasingly larger factor as the core distance increases. By contrast the mean energy carried by the muons (again distance dependent) is of the order of several GeV. The depth of the Haverah Park water-tanks was 3.2 radiation lengths so that the vast majority of the electromagnetic cascades is absorbed and detected. The detectors are thus very effective at measuring the energy flow in the shower disc.

The data described here relate to the lateral distribution function, $\rho(r)$, of the water-Čerenkov signal from 100 m to 2500 m from the shower axis. Accurate knowledge of $\rho(r)$ is essential to determination of the shower core (except in the rare cases where symmetry allows an independent determination) and the detailed dependence of $\rho(r)$ on distance and primary energy can be used to constrain the range of shower models and primary masses surmised at the energies in question. The lateral distribution is also of prime importance in the design of a giant array where the spacing of the detectors must be made large (> 1 km) if areas as great as 3000 km^2 are to be covered with economy.

2 Description of the Haverah Park Array

The Haverah Park array was located 220 m above sea-level at 53° 58.2′ N, 1° 38.2′ W. The detectors consisted of a number of units of varying area built from water-Čerenkov tank modules. The modules were of two types. The majority, which were used throughout the experiment, were galvanized iron tanks 2.29 m^2 in area and filled to a depth of 1.2 m with water brought from a nearby borehole in magnesium limestone. A minority of detectors were 1 $m^2 \times 1.2$ m deep water-Čerenkov detectors constructed from expanded plastic foam, 5 cm thick: these tanks were filled with de-ionized water as the plastic construction lacked the zinc of the galvanizing which is believed to have inhibited fungoid growth within the metal tanks. All tanks were lined with a white diffusing plastic (ICI Darvic). The water in each tank was viewed with an EMI photomultiplier (type 9618YB) with 5" diameter S11 Sb/CsO photocathode held so that it just dipped into the water. Detector areas larger than 2.29 m^2 were achieved by grouping together a number of the larger modules in huts with roofs having thicknesses less than 4 g cm^{-2}.

An arrangement of the detectors is shown in Figure 1 but not all of them operated throughout the lifetime of the project. Detectors A . . H were used over the period 1967–1987 while J, K and L (each 2.25 m^2) were added along with 21 $\times$ 1 m^2 units on a 150 m spaced grid in 1976. Detectors J, K and L were used until 1987 while the 21 detectors on the 150 m central grid were operated from July 1976 to June 1981. A description of the infilled section of the array, and the cross-calibration of its detectors with respect to the rest of the array, is given in Edge et al [6]. It was established that the systematic error in the calibration of individual detectors was less than 2% and that the fractional error $\frac{\Delta\rho}{\rho}$, in the measurement for water-Čerenkov density, ρ, was given by,

$$\left(\frac{\Delta\rho}{\rho}\right)^2 = 0.05^2 + \frac{0.4}{\rho A},$$

where A is the detector area (1, 2.25, 13.5 or 34 m^2).

The array was triggered whenever A1 (34 m^2) and any two of A2, A3 and A4 (also 34 m^2) recorded a signal above 0.3 equivalent muons m^{-2} with a resolving time of 4 μs. Data were then logged from the sets of detector units in various ways. Directions, except for some special investigations, were measured from the relative arrival times of the signals at A1 . . A4. The angular accuracy is well-described by $\sigma_\theta = 2.5° \sec\theta$ and $\sigma_\phi = 2.5° \csc\theta$. The primary energy was determined from a measurement of $\rho(600)$, the water-Čerenkov signal at 600 m from the shower axis [7,8].

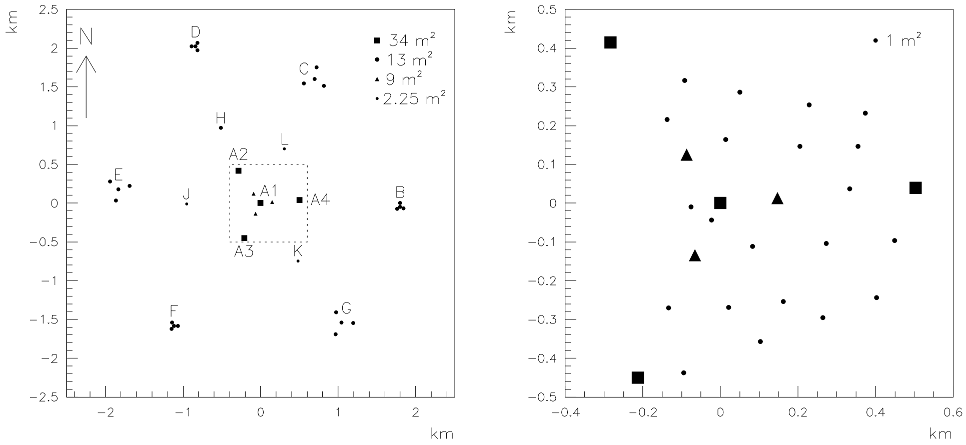

Fig. 1. Layout of the water-Čerenkov EAS array at Haverah Park. On the left the 12 km^2 array is shown; the key indicates the area of the various units. The area within the dotted box is shown expanded in the right plot where the positions of the 21 $\times$ 1 m^2 detectors of the infill array are indicated.

3 Measurement of the Water-Čerenkov Lateral Distribution

3.1 Early work on the lateral distribution of large showers

Arrays designed to measure the features of giant air showers have, for reasons of economy, detectors which are distributed at distances very much greater than the Molière radius, $\sim$ 75 meters at sea level. This has serious consequences for measuring shower features. On the one hand the fact that approximately half of the particles in the shower lie within one Molière unit of the shower axis has lead to methods of determining the shower energy which must rely on measurements at large axial distances such as the measurements of $\rho(600)$ used at Haverah Park [7]. On the other hand it is extremely difficult, from the available data, to determine the form of the lateral distribution accurately, and in particular the dependence of the function upon primary energy, which is relatively small. The change of lateral distribution function with zenith angle is much larger and relatively easy to observe.

In the earliest studies at Haverah Park a simple power law form, r^{-n}, was used to determine shower cores: n was found to depend strongly on zenith angle. As more detectors were added a lateral distribution function of the form,

$$\rho(r) = kr^{-\left(\eta + \frac{r}{r_0}\right)}, \tag{1}$$

was shown to fit the data adequately: η was found to vary with zenith angle and $r_0 = 4000$ m, independent of zenith angle: ρ is now used conventionally to represent a water-Čerenkov signal density. The normalization constant k was expressed as the resulting value of $\rho(600)$ for the η value being assumed or fitted; see section 3.4. Attempts to determine the variation of η with primary energy proved impossible. The difficulty is eloquently expressed by Linsley as arising from the danger of a "self-fulfilling prophecy" [9]. He pointed out that unless the core is determined by a sub-set of 3 or 6 detectors of a regular array and is hence obtainable independent of assumptions about the lateral distribution function, spurious dependences of shower parameters on shower size and hence on energy can arise. Indeed such problems were present in early attempts at Volcano Ranch and Haverah Park to find size dependencies but were largely solved once the difficulties were recognized and arrays of appropriate geometry were constructed. Relevant results for scintillator arrays have been reported by Linsley [9], by Diminstein et al [10], and by Yoshida et al [11].

3.2 Accurate measurements of the lateral distribution function at Haverah Park for $r < 700$ m

The most detailed studies of the lateral distribution function appropriate to the detectors used at Haverah Park were made while the 21 modules of 1 m^2 were in operation. The method used has been described elsewhere [12]. The final analysis [3] was based on 1424 events selected with primary energy $> 2 \times 10^{17}$ eV and zenith angle less than 45°. The lower energy limit was chosen so as to eliminate a bias which arises in the detection of showers of lesser energy where showers of steeper than average lateral distribution function (ie showers which have developed later in the atmosphere) are preferentially selected [13]. For each selected event the shower core was found using 3 or 4 detectors ringing the probable core position. The geometrical mean distance of the shower core from these "ringing" detectors was in the range $50 < r_n < 200$ m. Detailed empirical and Monte Carlo studies showed that the uncertainty in core position was close to $0.1r_n$. Only densities at distances $> r_n$ were used to determine the lateral distribution function. An iterative method was adopted to find the best value of η (in Equation 1) for each event using as a starting point the energy independent approximation for η available from earlier work,

$$\eta = 3.78 - 1.44 \sec\theta. \tag{2}$$

It was established that the final value of η, deduced after 6 iterations, was independent of whether 3.78 in Equation 2 was replaced by 3.48 or 4.08, at the level of better than 2% for final values of eta in the range $1.9 < \eta < 2.7$. A similar conclusion has been reached from Monte Carlo simulation studies. From this analysis [13] it was established that the average lateral distribution function was well described by the function of Equation 1. Specifically the average density ratios $\rho(50) : \rho(150) : \rho(500)$ are represented by the function of Equation 1 to better than 5%. In individual showers the ldf represents the ratios to an accuracy of better than 10% and the residual systematic error in the values found for η was less than 0.03.

A multiple linear regression was used to determine the dependence of η on zenith angle and energy [3]. The result is,

$$\eta = (2.20 \pm 0.01) - (1.29 \pm 0.05)(\sec\theta - 1) + (0.165 \pm 0.022)\log\left(\frac{E}{10^{17}\mathrm{eV}}\right). \quad (3)$$

This relation is an accurate average description of the lateral distribution function for showers with energies $2 \times 10^{17} < E < 4 \times 10^{18}$ eV, for the distance range $50 < r < 700$ m, and for zenith angles $\theta < 45^\circ$. The limited observing time sets the upper bound in energy for this analysis. It was also established that between-shower differences in η were significantly larger than expected from our understanding of the measurement uncertainties [12].

Although important for many purposes, the above study is restricted to showers which are smaller than those of most interest to the Auger project. An exploration of the applicability of the lateral distribution shape expressed by Equations 1 and 3 to larger showers is described below.

3.3 Extrapolation of the lateral distribution function to energies $> 4\times10^{18}$ eV, and to $r > 700$ m

Small numbers of extremely high energy showers ($> 10^{19}$ eV) fell close to or within the boundaries of the infilled array and measurements were available from all, or most, of the 21×1 m^2 detectors. In such events core location by simple arguments of symmetry is possible and the lateral structure is exceptionally well defined. Two such showers are shown in Figure 2; it is clear that the data are well described by a smooth monotonic function.

In these very large events meaningful particle density measurements were recorded at distances up to 2.5 km from the shower core. It became apparent that although the extrapolation of the ldf expressed by Equations 1 and 3 to $E \geq 10^{19}$ eV appeared to be valid at $r < 800$ m, at greater distances the observations were seriously under estimated. This can be clearly seen in Figure 2 where the predicted lateral distributions calculated on the basis of the above equations are plotted, together with the corrected ldf described below.

Since in conventional analysis of EAS array data each density measurement is assigned a weight proportional to its magnitude these very small, very distant, observations will have negligible effect in showers where several measurements

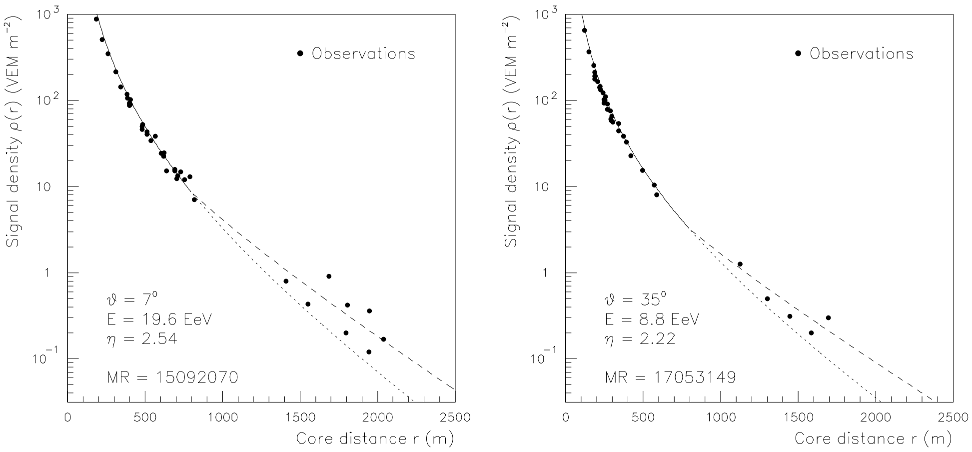

Fig. 2. Two extremely large EAS observed by the the Haverah Park 12 km^2 array in coincidence with the infill array. The reconstructed values of zenith angle and primary energy are indicated. At $r < 800$ m the predicted ldf calculated on the basis of Equations 1 and 3 is shown as a solid line. For $r > 800$ m the unmodified function is shown dotted, and the correction expressed by Equation 4 is dashed. (MR was an event code number which allowed the calculation of sidereal time.)

are available much closer to the core. However, this was not the case for the bulk of the Haverah Park data set at the highest energies ($> 10^{19}$ eV), and hence an investigation was conducted by one of us to determine a form of ldf which more accurately reproduced the observations at large distances [4]. The procedure followed was to select events each having several observations within $\approx$ 300 m of the core, analyze on the basis of the accepted ldf (Equation 1) using only those observations $<$ 800 m from the core, and then determine the observed:predicted ratio for each of the more distant measurements. The data were binned versus core distance, and a fit made to determine the average flattening of the ldf slope parameter η at core distances $>$ 800 m.

This work showed that the data could be well represented by flattening the established ldf for $r > 800$ m, while joining smoothly at 800 m, using the equation,

$$\rho(r) = k \left(\frac{1}{800}\right)^{\beta} r^{-\left(\eta + \frac{r}{r_0}\right) + \beta}, \tag{4}$$

where β was found to be 1.03 ± 0.05. For this work approximately 4 years of data taken whilst the central part of the array had the 150 m grid of detectors was used in the analysis. The extension of the previous ldf using Equation 4 is seen to be a good description of the experimental observations plotted in Figure 2. However due to the limited statistics available, and its somewhat "ad hoc" nature this parameterization of water-Čerenkov lateral distribution should be regarded as an approximation only.

3.4 Shower attenuation in the atmosphere, and the relation between ldf normalization and EAS primary energy

Using the argument that equal numbers of showers having given energy must be observed per unit solid angle at increasing zenith angle it is possible to investigate empirically the attenuation of EAS in the atmosphere. The data have traditionally been analyzed on the assumption of exponential behavior and a characteristic attenuation length λ derived. Work by Edge et al [14] showed that $\rho(600)$, the water-Čerenkov signal at 600 m from the shower axis, followed the relationship,

$$\rho_\lambda(600) = \rho(600) \exp\left(\frac{1018}{\lambda}(\sec\theta - 1)\right), \tag{5}$$

where $\rho_\lambda(600)$ is the equivalent value of $\rho(600)$ for a shower of equal energy arriving from the vertical direction: 1018 g cm^{-2} is the mean atmospheric depth of the array site, and θ the zenith angle of a given shower. The value determined for λ was 760 ± 40 g cm^{-2} derived from showers with zenith angles $< 60°$. No dependence of this value on increasing shower size could be detected up to $\approx 8 \times 10^{18}$ eV, although one might be expected when moving to the very highest energies ($> 10^{19}$). See [2] for a discussion.

The relationship between an EAS size parameter as measured by a ground array and the energy of the primary particle initiating the cascade must always be determined via shower model calculation whether analytic or Monte Carlo. Simulations by Hillas et al [7,8] indicated the relationship,

$$E = \beta \rho_\lambda(600)^\alpha, \tag{6}$$

where E is the energy of the primary particle in eV, $\beta = 7.04 \times 10^{17}$, and $\alpha = 1.018$. This was claimed to be valid for a wide variety of assumptions regarding interaction physics at energies beyond those probed by accelerator experiments, and also largely independent of the assumptions made regarding the mass of the primary particles. See [15] for a discussion.

4 Monte Carlo simulation of the Water-Čerenkov lateral distribution

Since the Auger experiment will be a hybrid system consisting of a conventional ground array operating in conjunction with a Fly's Eye style nitrogen fluorescence detector [16] the chosen sites will be at altitudes of ≈ 850 g cm^{-2}

overlying air mass. Sites closer to sea level ($\approx$ 1000 g cm^{-2}) generally lack the good conditions of atmospheric clarity required for the successful operation of sensitive optical instruments. The lateral distribution functions described in the previous section were derived for the altitude of Haverah Park (mean atmospheric depth of 1018 g cm^{-2}), and will be modified at greater altitudes. Additionally the ldf given above applies to water-Čerenkov detectors 1.2 m deep; modification is also to be expected for other depths.

It is important to realize that due to the large inter-detector spacing of the Auger ground arrays (1.5 km) it will extremely difficult to reconstruct events without prior knowledge of the lateral distribution shape. Hence to facilitate the extrapolation of the known lateral distribution functions to the new experiment it is extremely helpful to have a full shower and detector Monte Carlo simulation which cleanly reproduces the existing experimental results. Such a simulation is also very important for the project as a design tool allowing the experimental performance of proposed array configurations to be predicted and optimized. Fortunately the Auger experiment will be privileged with respect to previous EAS arrays in that the energy calibration of the system will not need to rely wholly on shower development calculations, but can be assessed experimentally through cross calibration with the nitrogen fluorescence detectors which measure event energy in a pseudo-calorimetric manner.

The MOCCA program developed by Hillas [17,18] is a sophisticated 4-dimensional cascade simulation incorporating a realistic atmospheric model, and implementing a “thin-sampling” algorithm which allows extreme-high energy showers to be run with reduced computation time. Output from this program is in the form of a list of particles reaching the ground giving type, position, energy, arrival time and incidence angle. A simulation of the Auger ground arrays has been written which takes as input the detailed information available from the MOCCA output and propagates individual particles through water-tank detectors[19]. Čerenkov light is radiated and ray-traced and the performance of the detector front-end electronics is accounted for. In the work described here the basic detector simulation element of this package has been used to predict the shape and normalization of the lateral distribution for EAS of 10^{19} eV, incident at various angles to the zenith, on an array of water-Čerenkov detectors 1.2 m deep. This was done for several primary cosmic ray types (proton, iron nucleus, gamma-ray).

The agreement between the lateral distribution shape predicted by these new simulations, and the old experimental parameterizations described in the previous section is illustrated in Figure 3. Note that the change in shape with increasing zenith angle of incidence is reproduced; the value of the parameter η calculated using Equation 3 shifts from 2.51 at $0°$ to 1.99 at $45°$; this is a large change but the Monte Carlo results track it closely. The attenuation of showers at increasing zenith angle and the conversion between ldf normaliza-

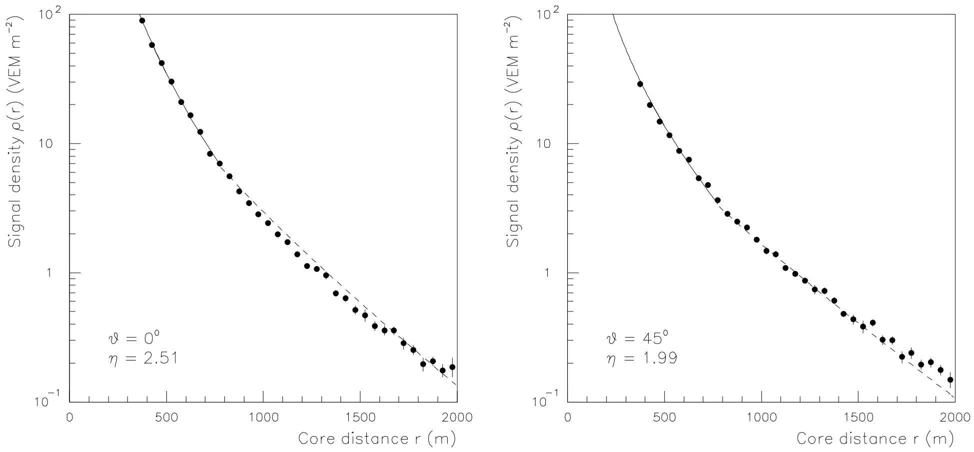

Fig. 3. Shower/detector Monte Carlo simulations of the lateral distribution of 10^{19} eV EAS as measured by an array of water-Čerenkov detectors 1.2 m deep. The points are the simulation results; the curves show the predicted form calculated using the experimental parameterizations Equations 1, 3, and 4. The value of zenith angle θ, and corresponding value of η are shown in each case. Note that the curves shown have been normalized to produce best fits in the region $400 < r < 800$ m.

tion and EAS energy predicted by Equations 5 and 6 are reproduced in this work to $\leq 30\%$ for zenith angles up to $\theta = 60°$: the latter simply indicates that the new simulations reproduce the results of the old to within that tolerance, and is true for the assumption of both proton and iron initiated showers. Currently the best agreement in shape is derived on the assumption that the primary particles at these energies are predominately heavy nuclei. However, this result is sufficiently sensitive to the details of the hadronic interaction models used by MOCCA at energies so far above constraint by accelerator data as to render it of little significance.

5 Conclusions

The measurements of the lateral structure of EAS as recorded by the array of water-Čerenkov detectors at Haverah Park have been described. Accurate experimental parameterizations expressing the mean ldf for $2 \times 10^{17} < E < 4 \times 10^{18}$ eV, $50 < r < 700$ m, and $\theta < 45°$ are given. Previously unpublished results concerning attempts to extrapolate these relations to the regime $E \geq 10^{19}$ eV and $r > 700$ m have been described: extrapolation in this energy domain appears valid, and an approximate correction term is given for the larger core distances. An equation expressing the attenuation of showers when observed at increasing zenith angle is quoted, and also the relationship between ground level density and primary energy as used in the Haverah Park experiment. These expressions collectively allow the shape and normalization of the lateral distribution at any zenith angle and energy to be predicted.

The results of recent Monte Carlo simulations of shower development and detector behavior are presented and compared to the complete parameterized ldf. The agreement is good and thus increases confidence that these calculations may be trusted as design tools for the Auger project, a proposed next generation detector system. Details of complete simulations of the ground array system, and predictions of its specific experimental performance will be the subject of a future article.

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The Haverah Park array was designed, constructed and operated through the efforts of too many individuals to list here — see [2]. A.M. Hillas is thanked for the use of the MOCCA EAS Monte Carlo, and H.D. Phuong for making the runs and tabulating the results used as input to the water-Čerenkov simulations described above. The computing division of the Fermi National Accelerator Laboratory, USA deserve thanks for the use of their facilities. Finally CLP and AAW would like to thank J.W. Cronin for organizing the Auger Detector Design Workshop at FNAL during the first half of 1995 where the simulation work described in this paper was carried out.

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